

## VALUATION TECHNOLOGIES AND RELATED APPLICATIONS

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### RELATED APPLICATIONS

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Application No. 60/030,085 of November 5, 1996;

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Application No. 60/117,261 of January 26, 1999;

Application No. 60/117,260 of January 26, 1999;

Application No. 60/127,512 of April 2, 1999.

### INCORPORATION BY REFERENCE

15

D’Zmura, David Andrew. “Forecasting Expectations of Insured Depository Default and Catastrophic Losses”. Proceedings of the IEEE/IAFE/INFORMS 1998 Conference on Computational Intelligence for Financial Engineering. CIFEr ’98. New York. P. 66-91. 1998.

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### STATEMENT

The inventions herein were not made under Federally sponsored research and development.

## TECHNICAL FIELD

Financial methods, processes, logic, algorithms, computerized apparatus and system.

## 5 SUMMARY

Methods and processes for valuing a financial security, wherein comprising unique mathematical and computational programming functions. A method for portfolio aggregation. Processes computing change in price of a security or portfolio respective a change in yield. These methods and processes are demonstrated as more robust and precise than standard art.

10 A security composed of similar securities, engineered in manufacture process to reflect target criterion. These securities would afford investors customized hedging or immunization needs. Business logic of analytic valuation, of security generation, and of arbitrage differentials and relative value spreads, such providing basis of computerized automation. A computer-based system which incorporates the business logic engines. A mutual fund, operating on the  
15 methods, processes, business logic and system, for investor public. Numerical data cleaning and preparation. Further process utilizing cleaned and prepared data. A process establishing a likelihood of default of depository banks by use of operating ratios. Method and process simulating variable statistical distributions in small sample environment. A theta variable modeling technology, including a process of theta's mathematical programming functions. An  
20 OAS/martingale valuation lattice, modified for default or loss and recovery or development. A business process, a swap transaction, between insured deposit default and catastrophic loss. Improvements to the art, and unique functional specifications, of computational calculators.

## BACKGROUND

The invention is necessitated by the shortcomings evident in prior art financial theories, methodologies, practices and products. The present invention affords improvements.

Prior art financial valuation rests on a pricing relation between coupon, yield to maturity and time to maturity, wherein the price is equal to a valuation formula based on a summation. This summation form creates a problem in derivation, as the first derivative, which should capture the change in price with respect to the yield, is actually only a first order term of the Taylor series approximation needed to find such a solution. Thus, the second derivative, which should capture the change in the change in price with respect to yield, is actually the second order term of the Taylor series approximation. Only by an infinite number of ordered terms does the solution become precise. The first derivative, duration, in the prior art, the first order term, is positive in magnitude, and hence fails to capture the negative magnitude of Einstein's fourth dimension, duration. Further, the prior art duration formulation contains four variables: coupon, yield, maturity and price, wherein price and yield are related by definition, and hence, the prior art formulation involves an analytic tautology, as price is defined as related to coupon, yield and maturity. Thus, prior art fails to deliver a precise algorithm for duration or to reflect space-time science. The present invention identifies a non-summation form and therefrom derives a precise first (and second) derivative.

When applied to valuation, and valuation changes over time, the prior art process is demonstrably imprecise, which often leads to tragic error when relied upon, as it is, in the hedging of securities and portfolios. And because the nature of imprecision can over or understate the actual change in pricing, when implemented within a portfolio comprising many securities, the error of each can become diffuse in the group and can lead to calamity.

The invention provides innovative algorithms for valuing financial securities, wherein the price (P) of a security relates to three variables via a function, said securities include the group of fixed-income, equity and premium policy instruments, said variables comprising the cash receipts (C), yield (Y) and time (T) to maturity or expiration, said function relating  
5 change in price with respect to yield, at instantaneous condition, by a novel formulation of duration. The invention's duration is a perfect form first derivative based on a non-summation formulation of price respective C, Y and T, and hence, is an exact conveyance of change in price with respect to Y in continuous time. The invention's duration is isomorphic to Einstein's postulation of the fourth dimension respective three dimensions of the space-time  
10 continuum, also called duration, which he shows as bearing negative magnitude. The invention's formulation of three variables, C, Y, T, embodies the characteristics of any financial security, and has continuous relation via a fourth dimension, duration, termed K.

In discrete time, the notion of theta, the derivative with respect to increments in time, is described and is implemented in valuation mechanics and in drawings and spreadsheets.

15 The processes accurately and precisely capture price change respective to time and yield.

The invention presents its governing yield which is the spot yield of a security or payment at present or future time. This notion can replace prior art forward rate curve process, this latter process bearing two serious flaws: 1) it is not a continuous curve, but a series of short hyperbolae strung together and joined where each asymptotically explodes; 2) the prior  
20 art forward rate curve is not isomorphic to or indicative of pricing realized forward.

The invention relates portfolio aggregation methodologies which afford the establishment of valuation and sensitivity values for a portfolio as a whole, useful in trading and hedging. A variable is specified, Yield M, useful for a portfolio of one or more securities.

The invention specifies algorithms, processes and systems which provide the computation of the novel financial methods. In addition, it specifies arbitrage based thereon, as well as a fixed-income mutual fund utilizing the relative value arbitrage afforded thereby.

The assortment of available financial instruments is limited, and often, what is  
5 available, with respect to a sought after duration (for instance, for immunization of a portfolio) may not be available in the market. To such ends, and as means for creating securities which may be alternatives or arbitrage matches for existing securities, the invention creates a new class of financial security, called a Replicated Equivalent Primary Security.

Most of the valuation methods and algorithms used in the pricing of options and  
10 derivatives rely on a set of assumptions regarding the log normal condition of the underlying variables. The invention provides data cleaning techniques to identify and to test for such log normal states of a variable, which are necessary as the conditions of the underlying change. The invention provides data analysis methods and process for small sample environments.

The data cleaning technology is applied to financial variables found in depository  
15 banking and P&C insurance industries. Such variables, having nominal value, can function as the underlying variables of financial securities modeled thereon, and therefore, the invention organizes and describes the technology pertaining to underlying state, theta, variables. The invention further specifies a process useful in establishing the likelihood of default of insured depository banks. As the insured banks pay a fee to the FDIC Bank Insurance Fund, such  
20 process, utilizing a set of operating ratios in concert, can be helpful in identifying the causes of default risk, as well as assessing the level of risk on an aggregate or individual bank basis. Included in modeling a theta variable is described its mathematical programming functions.

Pursuant to the mechanics of prior art OAS (Option Adjusted Spread) and martingale valuation lattice, useful for modeling values having an element of default or loss probability, is the shortcoming that default or loss is held realized at the event of default (or catastrophe) whereas default is followed by recovery, and a catastrophic event by loss development. Hence,  
5 the invention specifies a modified lattice incorporating the recovery and development.

Given that the insured depository banks pay insurance fees to the FDIC's BIF, and given that the Property & Casualty insurance industry has a shortage of underwriting capacity with respect to the prospect of severe catastrophic losses, and because the taxpaying public stands as the end guarantor against catastrophic bank depository default and catastrophic loss,  
10 presented herein is a swap transaction between the quasi-governmental bodies of the FDIC and the NAIC, between insured depository default and catastrophic loss, useful on industry treaty or per individual institution basis. Specified therein is the use of public sector capital. Such swap or treaty reinsurance provides a mechanism towards open market reinsurance of such relatively uncorrelated risks between the insured banking and P&C insurance industries.

15 Among the aspects of the invention are specifications of computerized apparatuses and systems to performing valuation, analysis, identification and execution of transactions. Further aspects of the invention include improvements to the art of computational calculators. Such improvements include resident educational features for teaching and scholastic usage. Specified in functional detail is a financial engineering calculator, with computational and  
20 resident coded features suitable to the demanding needs of the technical financial community. To date, the prior art financial calculators provide only few rudimentary resident algorithms and lack resident reference resource type items, these shortcomings directly addressed herein.

## BRIEF DESCRIPTION OF THE DRAWINGS

The formulaic means and functions for valuation of a security: Figure 1, Formula 1.1; Figures 2 and 3, Formula 1.2 and alternate 1.2d; Figure 4, Formula 1.3; Figure 5, Formula 1.4. The means and functions to aggregate a portfolio or divisible security, Figures 6 to 10. The process of valuation, of establishing pricing sensitivities, and of estimating change in price, based on means, Figure 11. The method of portfolio aggregation based on means, Figure 12. Figure 13, a sample portfolio comprised of financial securities, with data values per security. Figures 14 and 15, the aggregation programming functions and aggregate values generated. Figure 16, Yield M values and relative basis to zero spot corresponding to portfolio maturity. Figure 17, estimation over time of change in price by change in Yield M, vs. standard Yield. Figure 18, the pricing sensitivity of duration, K, contrasted with standard duration. Figure 19, standard convexity, used in combination with K and Yield M, estimating the change in price. Figure 20, the convexity V, varied inputs, contrasted with standard; Yield M-spot differential. Figure 21, estimation of price change based on Yield M, K and V, see Figures 1 through 22. Figure 22, means and functions for estimating change in price, operating by Figures 1 to 22. Figure 23, demonstrated mathematical relation of duration and convexity to change in price. Figure 24, the method of manufacturing valuation and pricing sensitivity data for a portfolio. Figures 25, coded algorithms corresponding to formulae for valuation of security or portfolio. Figures 26, coded algorithms corresponding to formulae for valuation of security or portfolio. Figure 27, means and functions for estimating price change, plus derivative respective time. Figure 28, means and functions for estimating price change, after Figure 27, for a portfolio. Figures 29 and 30 implement Figures 27 and 28, and calculate and sort arbitrage differentials. Figure 31 diagrams analytic valuation engine; Figure 32, engine modified for time derivative.

Figure 33 diagrams automated arbitrage engine, spread arbitrage; Figure 34, notch arbitrage.

Figure 35 process manufacturing a REPS security. Figures 36, 37, 38, three alternate REPS.

Figure 39, the target security. Figure 40, price matrix, buy and sell prices of REPS and target.

Figure 41, arbitrage opportunities between buy, sell pairings, each of REPS and target; sorted.

5 Figure 42, REPS generator. Figure 43, the automated arbitrage engine, basis REPS arbitrage.

Figure 44, systems architecture, integrating the core valuation, REPS and arbitrage engines.

Figure 45, method of mutual fund operation, incorporating valuation, REPS and arbitrage.

Figure 46 graphically renders nominal data of variables of depository default and P&C losses.

Figure 47, tabled arrays of variables' nominal and inflation-adjustment data; inflation scalar.

10 Figure 48 is tabular rendering of variables' log and delta log data arrays using adjusted data.

Figure 49 graphically renders adjusted data of variables of depository default and P&C losses.

Figure 50 graphically renders log data of the variables of depository default and P&C losses.

Figure 51 graphically renders log delta data of variables of depository default and P&C losses.

Figures 52 through 54 render log and delta log data of deposit closings and of deposit losses.

15 Figures 55 through 58 graphically render processed P&C underwriting and catastrophe data.

Figure 59 graphically renders variable statutory underwriting loss respective catastrophe loss.

Figure 60 graphical rendering, log and delta log values for variable insured catastrophe loss.

Figure 61 graphical rendering, log and delta log values of underwriting and catastrophe losses.

Figure 62, descriptive statistics of deposit closings, nominal, adjusted, log and log delta data.

20 Figure 63, descriptive statistics of catastrophe loss, nominal, adjusted, log and log delta data.

Figure 64, descriptive statistics of deposit closings and catastrophe loss, over a recent period.

Figure 65, descriptive statistics of deposit closings and catastrophe loss, over life of variable.

Figures 66, 67 and 68, histograms of deposit closings and catastrophe loss, in varied periods.



Figure 69 growth scalar; growth-adjusted nominal, inflation-adjusted, log and delta log data of deposit closings. Figure 70, descriptive statistics of deposit closings' growth-adjusted data. Figure 71, growth scalar; growth-adjusted nominal, inflation-adjusted, log and delta log data of catastrophe losses. Figure 72, histograms of growth-adjusted deposit closings and cat loss.

5 Figure 73, data arrays of U.S. insured deposits and assets closed and total deposits and assets. Figure 74, default magnitude data of closed deposits and assets per total deposits and assets. Figure 75, graphical rendering of default magnitude data of closed deposits to total deposits. Figure 76, descriptive statistics and histograms of default data of closed deposits and assets. Figure 77, operating data of total deposits (TD), interest-bearing deposits (IBD), total assets

10 (TA), interest-bearing assets (IBA) and interest bearing liabilities (IBL). Figure 78, operating ratios of TD/TA, IBD/IBA, IBD/IBL, TD/IBA, TD/IBL and IBA/IBL. Figure 79 graphically renders the ratios of TD/TA, IBD/IBA, IBD/IBL and TD/IBA. Figure 80 graphically renders the ratios of TD/IBA, TD/IBL and IBA/IBL. Figure 81, graphical rendering of the historical relation of the ratios TD/TL, TD/IBL and IBA/IBL to deposit closings as default martingale.

15 Figure 82, a method for cleaning and preparing nominally valued data plus further processing. Figures 83 through 89, processing steps of small sample technology and representative results. Figure 90, alternate process step of small sample process and representative sequence results. Figure 91, mathematical programming functions modeling a theta, underlying state, variable. Figure 92, the mathematical programming functions for modeling a security based on a theta.

20 Figure 93, a martingale valuation lattice, modified for default/loss and recovery/development. Figure 94, a swap transaction diagrammed between depository default and catastrophic loss. Figures 95 through 99, improvements to a calculator; Figure 100, a financial engineering unit.

## DESCRIPTION OF THE PREFERRED EMBODIMENTS

A process for the manufacture of financial data using the endogenous variables of a financial security, and for estimating change in the security's prices given change in its yield with respect to time, which comprises implementing within conformance to the Formula 1.1:

1.1  $P = f \{ C, Y, T \}$  where C, Y, and T are variables endogenous to the security

P = Price

C = Cash Receipts, periodic coupon, dividend or premium payments

Y = Yield, a single term relating security's return, relative to P, C, T

T = Time, a terminal or continuous measure of the life of the security.

The Formula 1.1 provides a methodological process engineering the data values of a security's governing yield. Y, governing yield, establishes market rates of return, showing relativity appropriate the market yield structure, a zero spot rate for respective security's time.

The Formula 1.2 or 1.2d are applied to numerically determine governing yield, for a single security issue, and a portfolio of issues, or for a basket of divisible cash receipts:

1.2 
$$\text{Yield M} = \frac{\sum (\text{Maturity} \times \text{Portfolio Coefficient} \times \text{YTM}), \text{ for all issues}}{\sum (\text{Maturity} \times \text{Portfolio Coefficient}), \text{ for all issues}}$$

where Yield M = Governing Yield = Y

Maturity = Time = Maturity in Years

Portfolio Coefficient = Present Value, per issue/Present Value,  $\sum$  issues

Present Value = Accrued Interest + (best bid Price  $\times$  Face Value)

YTM = Yield-To-Maturity, a means providing yield respective time;

for Single Issue: its Portfolio Coefficient is one, its Yield M = its YTM

for Portfolio: the formula creates a single Yield M value of all issues.

1.2d 
$$\text{Yield Md} = \frac{\sum (\text{Duration} \times \text{Portfolio Coefficient} \times \text{YTM}), \text{ for all issues}}{\sum (\text{Duration} \times \text{Portfolio Coefficient}), \text{ for all issues.}}$$

Yield M, the governing yield, is the Yield-To-Maturity of a fixed-income security, such as U.S. Treasuries, but is not equivalent to standard YTM forms on a portfolio basis.

Formula S.1 and S.2 provide formula algorithms for semi-annual-coupon Yield-To-Maturity:

$$5 \quad S.1 \quad \text{Price} = \frac{C}{2} \sum_{T=1}^{2T} (1 + Y/2)^{-T} + (1 + Y/2)^{-2T}$$

where C = Coupon    Y = YTM    T = Maturity (in years).

10        An alternate expression of the price, yield relation, without summation, Formula S.2:

$$S.2 \quad \text{Price} = \frac{C}{Y} (1 - (1 + Y/2)^{-2T}) + (1 + Y/2)^{-2T}$$

where C = Coupon    Y = YTM    T = Maturity (in years).

15

However, the processing algorithms, S.1 and S.2 are not identical and do not provide exactly equivalent data for the same security, each algorithm using identical C, Y, and T data. This genuine difference is evidenced in my data processing with this algorithmic function. These two algorithms, for solving YTM, stand apart, yet comprise a relative value correlation.

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Because these two algorithmic measures of the yield-to-maturity differ, so too do their mathematical derivatives, the first derivative, or, duration, ie. Duration, modified annualized:

$$25 \quad (S.1') \quad (Duration) \quad \frac{\frac{C}{Y^2} \left[ 1 - \frac{1}{(1 + Y/2)^{2T}} \right] + \frac{2T(100 - C/Y)}{(1 + Y/2)^{2T+1}}}{2P} \quad \text{where } D = \Delta P / \Delta YTM$$

Y = YTM  
T = Mat. in Years  
C = Coupon  
P = Price (par=100).

S.3    Durmodan =

(S.2') (Duration, modified annualized, semi-annual C)

$$30 \quad 1.3 \quad K = \frac{-C}{Y^2} (1 - (1 + Y/2)^{-2T}) + \frac{C}{Y} (T + TY/2)^{-2T-1} - (T + TY/2)^{-2T-1}$$

wherein I utilized the chain rule of calculus to derive K, first derivative of S.2

where C=Coupon    Y=YTM    T=Maturity in Years     $\delta Y = \Delta \text{Yield M}$      $\delta P = \Delta \text{Price}$   
(decimal entry, portfolio)    (portfolio)    (with  $\delta P / \delta Y = K$ )

35

$$1.3n \quad K \text{ generalized} = \frac{-C}{Y^2} (1 - (1 + Y/n)^{-nT}) + \frac{C}{Y} (T + TY/n)^{-nT-1} - (T + TY/n)^{-nT-1}$$

wherein  $n = \#$  coupon or dividend payments per annum, wherein 1.3 can be written:

(S.2') (Duration, mod. ann., semi-ann.)

$$1.3w \quad K = \frac{-C}{Y^2} + \frac{C}{Y^2} (1 + Y/2)^{-2T} - (1 - C/Y)(T + TY/2)^{-2T-1}$$

The Formula 1.3 is in noteworthy contrast to Formula S.3, as the former relies only on C, Y and T to relate a security's change in price respective its change in yield-to-maturity. It is a clean, non-tautological expression of duration, as it does not simultaneously require price, which, in the implementation of S.3, is an exogenous, market-generated, variable. Also, K is of inverse sign to the conventional duration, D, but not its inverse, K accurately reflecting the fact that as the yield of a security rises, its price declines, *ceretis paribus*. K states duration sensitivity to change in price respective change in governing yield. Hence, K depicts the primary pricing sensitivity of a security, or portfolio thereof, and satisfies the Formula 1.1.

The second derivative, convexity, the price sensitivity indicating change in curvature of the first derivative, that is, how the security's propensity to change in price is transforming:

$$(S.1'') \text{ (Convexity)} \quad \frac{2C}{Y^3} \left[ 1 - \frac{1}{(1+Y)^{2T}} \right] + \frac{2C(2T)}{Y^2 (1+Y)^{2T+2}} + \frac{2T(2T+1)(100 - C/Y)}{(1+Y)^{2T+2}}$$

$$S.4 \quad \text{Convex} = \frac{\quad}{4P}$$

$$(S.2'') \text{ (Convexity)} \quad 1.4 \quad V = \frac{2C}{Y^3} - \frac{\frac{2C}{Y^3}}{(1+Y/2)^{2T}} - \frac{\frac{CT}{Y^2}}{(1+Y/2)^{2T+1}} - \frac{\frac{C}{Y^2}}{(T+TY/2)^{2T+1}} + \frac{(1+C/Y)(T^2+T/2)}{(T+TY/2)^{2T+2}}$$

wherein I utilized the chain rule of calculus to derive V,  $V = K'$  and  $V = S.2''$

wherein  $V \neq$  standard Convexity (Convex);

wherein V can be calculated  $Y =$  standard YTM, Yield M, or, Yield M – YTM basis.

A method for valuing a security by its endogenous variables, comprising steps of:  
 identifying the data values for the security's endogenous variables, of C, Y, T, per 1.1;  
 establishing Yield M, means for performing process 1.2, or using spot or quote values;  
 utilizing values of C, Yield M, T, calculating the security's price, if it is fixed-income:  
 5                   by solving price, means for performing either S.1 or S.2, or both separately;  
 utilizing values of C, Yield M, T, calculating duration and convexity price sensitivity:  
                   by solving duration, means for performing S.3 or 1.3, respective S.1 or S.2;  
                   by solving convexity, means for performing S.4 or 1.4, respective S.1 or S.2.

10           A method for valuing a financial portfolio, containing more than one divisible issue,  
 by singular portfolio (P) data values of endogenous variables  $C^P$ ,  $Y^P$ ,  $T^P$ , comprises steps of:  
 identifying the data values for each issue's endogenous variables of C, Y, T, per 1.1;  
 generating the portfolio coefficients for each single security issue in a portfolio, by:  
                   1.5     Portfolio Coefficient, per each Issue =  $\text{Present Value}^I / \text{Present Value}^P$ ;  
 15                   1.5a    $\text{Present Value}^I = (AI + (\text{Bid Price} \times \text{Face Value}))$ , per Issue (I);  
                   1.5b    $\text{Present Value}^P = \sum (AI + (\text{Bid Price} \times \text{Face Value}))$ , for all Issues;

generating aggregate portfolio (P) data relating a portfolio's aggregate values, by:

- 1.6a    $\text{Present Value}^P = \sum (AI + (\text{Bid Price} \times \text{Face Value}))$ , for all Issues;  
 1.6b    $\text{Accrued Interest}^P = \sum \text{Accrued Interest, AI}$ , for all Issues;  
 20       1.6c    $\text{Face Value}^P = \sum \text{Face Value}$ , for all Issues;  
           1.6d    $\text{Implied Price}^P = (\text{Present Value}^P - AI^P) / \sum \text{Face Value}$  for all Issues;

generating aggregate portfolio (P) data relating portfolio's endogenous variables:

$$1.7a \quad C^P = \text{Cash Flow}^P = \sum C \times \text{Portfolio Coefficient, for all Issues;}$$

$$1.7b \quad T^P = \text{Time}^P = \sum \text{Maturity} \times \text{Portfolio Coefficient, for all Issues;}$$

$$1.7c \quad Y^P = \text{Yield}^P = \sum \text{Yield} \times \text{Portfolio Coefficient, for all Issues;}$$

if for a portfolio of U. S. Treasury issues, the formulations of  $C^P$ ,  $Y^P$ ,  $T^P$  are:

$$5 \quad 1.8a \quad C^P = \text{Coupon}^P = \sum \text{Coupon} \times \text{Portfolio Coefficient, for all Issues;}$$

$$1.8b \quad T^P = \text{Maturity}^P = \sum \text{Maturity} \times \text{Portfolio Coefficient, for all Issues;}$$

$$1.8c \quad Y^P = \text{Yield}^P = \sum \text{Yield} \times \text{Portfolio Coefficient, for all Issues}$$

where Yield by Yield M, by zero spot for T, or by YTM of S.1 or S.2;

processing C, Y, T, per issue, to generating portfolio's duration and convexity:

$$10 \quad 1.9a \quad \text{Duration}^P = \sum \text{Duration} \times \text{Portfolio Coefficient, for all Issues;}$$

$$1.9b \quad \text{Convexity}^P = \sum \text{Convexity} \times \text{Portfolio Coefficient, for all Issues.}$$

or utilizing portfolio values,  $C^P$ ,  $Y^P$ ,  $T^P$ , calculating

Duration, means for performing S.1' or 1.3, respective S.1 or S.2;

Convexity, means for performing S.1'' or 1.4, respective S.1 or S.2;

15 establishing Yield M, means for performing process 1.2, or using spot or quote Y.

The Figure 1 presents Formula 1.1. The Figure 2 presents Formulae 1.2. The Figure 3 presents Formula 1.2d. The Figure 4 presents Formula 1.3. The Figure 5 presents Formula 1.4. The Figure 6 presents Formulae 1.5, 1.5a and 1.5b. The Figure 7 presents Formulae 1.6a, 1.6b and 1.6c. The Figure 8 presents Formulae 1.7a, 1.7b and 1.7c. The Figure 9 presents Formulae 1.8a, 1.8b and 1.8c. The Figure 10 presents Formulae 1.9a and 1.9b. The Figure 11 presents a process manufacturing financial data. The Figure 12, steps of method for valuing a portfolio.

A test portfolio of U.S. Treasuries is used to demonstrate operation of algorithmic formulae in processes and within methods, and to test the efficacy of processes and methods. The example portfolio is comprised of seven U.S. Treasury issues, maturing in five-year span, comprising a ladder portfolio, with each issue spread in maturity along the yield curve's time.

5 The issues are U.S. Treasury Notes, paying semi-annual coupons until maturity. This class of financial security is simple in composition and variables, transparent in liquidity, and has a price-efficient market yield structure; the market yield of a term is its zero spot rate, this discovered by the zero spot yield curve, composed of U.S. Treasury zero-coupon STRIPS. I designed, conducted and implemented this test implementation in Spring and Summer 1996,

10 and test three dates, of 3/22/96, 4/3/96 and 4/25/96, were utilized to set and determine values. The Issue 2) was held as purchased on 3/22/96, hence its absence of accrued interest that date. Not knowing the issuance day of month, I held each Note to mature on the 15<sup>th</sup> of the month. A coupon payment from Issue 5), actually maturing 3/31/99, was held as paid out on 3/15/96.

The Figure 13 identifies data for each of the Treasury issues: by its calendar date of maturity; and by its coupon interest, these fixed through the issue's life. Key values for each

15 issue at each of the three test dates are presented: years to maturity; ask yield; bid price; accrued interest; and full value (present value), wherein the yield and price data were culled from published financial newspaper data, and wherein the other data were calculated per date, specifically *The Wall Street Journal* and *Investor's Daily*. The ask yield data, contained in the

20 newspapers, operate, and are generated, by Yield-To-Maturity Formula S.1 on ask prices.

Utilizing the Formulae and method of portfolio aggregation, the Figure 14 presents the aggregate value calculations for a portfolio as a whole, whereas the Figure 15 presents the aggregate values for this portfolio, being manufactured data for the portfolio at each test date.

Having established the maturity in years for the portfolio at each test date, isomorphic to an implied maturity at future date, calculated from test date, counting out maturity in years. For that implied future date, the zero spot market rate was approximated from U.S. STRIPS. These data are presented in Figure 16, as well as the portfolio's Yield M and YTM values. For each of the three test dates, the portfolio's Yield M better approximated the Zero Spot, than did the portfolio YTM. The portfolio's Yield D appeared to best approximate Zero Spot.

To conduct further testing of the processing algorithms, and to implement the methods on the example test portfolio, the estimation of a change in price for a given change in yield, the duration/convexity factorization, where the actual change in yield is given between dates. The standard computational process for estimating actual change in price for change in yield, comprises the summation of A) and B), this equivalent to C), due to duration and convexity:

S.5            A)      $\Delta \text{ Price, due to Duration} = (-\text{Duration mod. ann., e.g. S.3}) \times \Delta \text{ YTM}$

B)      $\Delta \text{ Price, due to Convexity} = \frac{1}{2} \times \text{Convexity, e.g. S.4}) \times (\Delta \text{ YTM})^2$ :

15            C)      $\Delta \text{ Price, due to Duration and Convexity} = \text{A) + B):}$

where values for standard Duration and Convexity at beginning of time-frame.

The Figure 17 presents the actual change in yield between the test dates, as well as the portfolio duration and convexity, and utilizing the duration, convexity factorization, estimating change in price,  $\Delta \text{ Price}$ , and after calculating the actual  $\Delta \text{ Price}$  of the portfolio, the percentage accuracy of the factorization, as based upon the various formulations of yield. The Yield M data evidence a robust estimation of the actual change in portfolio implied price, whereas the YTM data did not closely estimate the actual change in price for the time-frames.



The Figure 18 presents the calculation of duration variables, standard (Formula S.3) and K (Formula 1.3) for the aggregate portfolio, utilizing the coupon, maturity and YTM (and Price, for implementing standard) of the aggregate portfolio, showing their data's difference.

To implementing a duration, convexity factorization, wherein utilizing K for duration, formulation recognizing K as of inverse sign to Durmodan, thus,  $\Delta$  Price, due to Duration (K):

$$1.10k \quad A) \quad \Delta \text{ Price, due to Duration (K)} = K \times \Delta Y.$$

The Figure 19 processes 1.10k, the manufactured data for the portfolio, of its K value, Duration, and  $\Delta Y$ , this latter term herein represented in the drawing by theoretical value  $\delta Y$ .

The accuracy of the estimate for  $\Delta$  Price, using K for the Duration value, and  $\delta Y$ ,  $K = \delta P / \delta Y$ , as expected by the relation  $\delta P \cong \Delta \text{ Price}$ , returns highly accurate estimates, with Error < 0.5%.

To implementing a duration, convexity factorization, utilizing V for convexity,

$$1.10v \quad B) \quad \Delta \text{ Price, due to Convexity (V)} = \frac{1}{2} \times V \times (\Delta Y)^2.$$

The Figure 20 processes 1.10v, V values for portfolio, on Yield or spread basis for Y.

The Figure 21 implements a price/yield factorization containing  $\delta Y$ , K and V, comprising Figure 22: the “duration/convexity” factorization, the Formula 1.10, incorporating 1.10k and 1.10v, being means for estimating a change in price respective yield through time:

$$1.10 \quad \text{Estimated } \Delta \text{ Price} = (K \times \delta Y) + \left( \frac{1}{2} \times V \times (\delta Y)^2 \right)$$

where  $\delta Y = \Delta Y = \Delta \text{Yield M}$ ; approximated  $\Delta$  zero spot, or  $\Delta \text{ Price}/K$

$K = \text{Duration}$ , e.g. Formula 1.3 and  $V = \text{Convexity}$ , e.g. Formula 1.4.

The nature of price, of price as defined by market price setting, is, at core, discrete, being that an outcry system of bids and asks has transactions marking prices and offer basis, with a particular price or quote existing at a particular time. The market's price is marked by discrete transactions at discrete moments. While open markets have standing pricing quotes, they are like held notes in musical play, momentary, having a duration for a snippet of time. Hence, the exogenous variable of Market Price is temporally represented as in discrete form.

The nature of yield, of yield with respect to time, hence, to the existence of continuity, is best understood as continuous, an on-going time value of money, without gap or stoppage. As an endogenous variable, yield provides the inner valued material of a security's existence, being calculable at any discrete moment bearing pricing data, and is reflected in pricing data. The nature of price discovery is by transaction point in time, standing quote snippet of time, and by the continuously unfolding, on-going fabric of developing information, events, rumors.

Because yield is calculable at any discrete time or date for which pricing data exists, it, like time, has a dual nature, that is, it can be reduced and identified at or in discrete time. The endogenous variable,  $K$ , an algorithm for calculating a security's duration, duration being a constant of the security throughout its life which transforms with respect to change in yield, used to establish a security's hedge ratio, value of basis point, or point of immunization ( $-K$ ). Duration is an instantaneous constant, but it can also estimate change between points in time. The algorithm,  $K$ , expresses a negative number, as a decrease in duration causes price to fall.

Respective the magnitude of change reflected in pricing, yield or duration, the breadth and deviation of convexity, conveying the impetus and force of the change to change in yield, is much greater, though its discrete contribution to changes in yield and price, is much less.

The generalized form of Formula 1.10, of its mathematical programming function, in Figure 23, function estimating the change in a security's price, with respect to yield and time:

$$1.11 \quad \Delta \text{ Price} = ( - | \text{Duration} | \times \delta Y ) + ( \frac{1}{2} \times \text{Convexity} \times (\delta Y)^2 )$$

5 where  $\delta Y \cong \Delta Y = \Delta \text{Yield M}$ ; instantaneous, or across points in time

Duration = Formula 1.3 or S.3 and Convexity = Formula 1.4 or S.4.

A method of estimating change in the price of a financial security, the Figure 24, such price function of said financial security determined by identified endogenous variables, satisfying a mathematical pricing function, Formula 1.1, steps of method conforming thereto. The values of endogenous variables, C, Y, and T, wherein if a fixed-income security with semi-annual interest coupons, these variables conform to its coupon rate, yield-to-maturity and years-to-maturity, respectively, and endogenous variable, price, are identified for security, with such data values determined at a given point in time, or by expectation or simulation.

15 For a portfolio of securities, the portfolio is comprised of each security, each with its own data values for C, Y, T; aggregate data values of the portfolio, Formulae 1.5 through 1.9.

Processing data from C, Y and T, generating data value of governing yield variable, Yield M, the Formula 1.2, or variable of Yield Md, the Formula 1.2d, for security or portfolio. Providing said data of said endogenous variables to database, storage, or further processing.

20 Determining the security or portfolio data values for duration and convexity, include the group of paired mathematical functions, Formulae S.3 and S.4, or Formulae 1.3 and 1.4.

Determining change in price for change in Yield M, Formula 1.10 or Formula 1.11.

Means for processing and generating data by Formulae S.2, S.3, S.4, the Figures 25:

Relation of Price to Yield, with respect to time:

S.2c semi-annual  $P = PR = \frac{((C/Y)*(1-(1+(Y/2))^{(-2*T))})+(1+(Y/2))^{(-2*T)}}{1}$   
where C, Y and P are decimal values, T=Maturity in years

5 S.2cn generalized  $P = PRBOND = \frac{((C/Y)*(1-(1+(Y/N))^{(-N*T))})+(1+(Y/N))^{(-N*T)}}{1}$   
where N=n= cash receipts per annum, e.g. semi-annual=2

Duration, First Derivative Sensitivity of Price to Yield, incl. variable Price Data:

10 S.3c semi-annual  $Durmodan=DURMOD = \frac{(((C/2)/((Y/2)^2))*(1-(1/((1+(Y/2))^{(2*T)}))))}{+((2*T)*(100-((C/2)/(Y/2))))/((1+(Y/2))^{(2*T)+1}))/((2*P))}$   
where P = Price (of 100)

S.3cn generalized  $Durmodan=DURMD = \frac{(((C/N)/((Y/N)^2))*(1-(1/((1+(Y/N))^{(N*T)}))))}{+((N*T)*(100-((C/N)/(Y/N))))/((1+(Y/N))^{(N*T)+1}))/((2*P))}$   
where N=n= # C periods per annum, e.g. semi-annual=2; T=Maturity in years

15 Convexity, Second Derivative Sensitivity of Price to Yield, incl. variable Price Data:

S.4c semi-annual  $Convex = CON = \frac{(((C/((Y/2)^3))*(1-(1/((1+(Y/2))^{(2*T)}))))}{-((C*(2*T))/((Y/2)^2)*((1+(Y/2))^{(2*T)+1})))}{+(((2*T)*((2*T)+1)*(100-(C/Y)))/((1+(Y/2))^{(2*T)+2}))/((4*P))}$

20 S.4cn generalized  $Convex = CONDP = \frac{(((C/((Y/N)^3))*(1-(1/((1+(Y/N))^{(N*T)}))))}{-((C*(N*T))/((Y/N)^2)*((1+(Y/N))^{(N*T)+1})))}{+(((N*T)*((N*T)+1)*(100-(C/Y)))/((1+(Y/N))^{(N*T)+2}))/((4*P))}$

Duration (S.3), Convexity (S.4) Factorization, Change of Price to change of Yield:

25 S.5c generalized  $\Delta P = DP = - (Durmodan)*(CHY) + (0.5*Convexity)*(CHY^2)$   
where  $CHY = \delta Y = \Delta Y = (Y_1 - Y_0)$ ,  $Y_0 = Y$  at start,  $Y_1 = Y$  at second point  
and where  $\Delta P = DP = -abs(DurationS.3cn)*(CHY) + (0.5*(ConvexityS.4cn)*(CHY^2))$ .

30 Means for processing and generating data by Formulae 1.2, 1.3, 1.4, 1.10, Figures 26:

1.2 Yield M = YM =  $\frac{(\sum\{(Maturity*Portfolio\ Coefficient*YTM)_1, (M*PC*YTM)_{2,...}\})}{(\sum\{(Maturity*Portfolio\ Coefficient)_1, (M*PC)_{2,...}\})}$

35 1.2d Yield Md = YMD =  $\frac{(\sum\{(Duration*PC*YTM)_1, (D*PC*YTM)_{2,...}\})}{(\sum\{(Duration*Portfolio\ Coefficient)_1, (D*PC)_{2,...}\})}$

Duration, (1.3), First Derivative, Price Sensitivity to Yield, endogenous C, Y, T, only:

semi-annual

$$1.3cw \quad K = DPDY = \frac{(-C/(Y^2)) * (1 - ((1 + (.5 * Y))^{(-2 * T)}))}{((C/Y) * ((T + (.5 * Y * T))^{((-2 * T) - 1)})) - ((T + (.5 * Y * T))^{((-2 * T) - 1)})}$$

where C and Y are decimal values, T=Maturity in years

generalized

$$1.3cn \quad K = BONK = \frac{((-C/(Y^2)) * (1 - ((1 + (Y/N))^{(-N * T)}))}{(((C/Y) - 1) * T * ((1 + (Y/N))^{((-N * T) - 1)}))}$$

where C and Y are decimal values; N=n #C periods per annum; T=Maturity in years

Convexity, (1.4), Second Derivative Sensitivity of Price to Yield, endogenous Data only:

generalized

$$1.4cn \quad V = BONV = \frac{(((2 * C)/(Y^3)) * (1 - (1 + (Y/N))^{(-N * T)})) - ((C/Y^2) * (2 * T) * ((1 + (Y/N))^{((-N * T) - 1)})) - (((C/Y) - 1) * ((N * T) + 1) * (T/N) * ((1 + (Y/N))^{((-N * T) - 2)}))}{10000}$$

where C and Y are decimal values; N=n #C periods per annum; T=Maturity in years

spread-based, semi-annual

$$1.4cv \quad V = VEXA = \frac{(((2 * C)/(Y^3)) - (((2 * C)/(Y^3)) * ((1 + (Y/2))^{(-2 * T)})) - ((C * T)/(Y^2)) * ((1 + (Y/2))^{((-2 * T) - 1)}) - ((C/(Y^2)) * ((T + (T * (Y/2)))^{((-2 * T) - 1)})) + (((1 + (C/Y)) * ((T^2) + (T/2)) * ((T + (T * (Y/2)))^{((-2 * T) - 2)})))/10000}$$

where e.g. Y=spread=YieldM-YTM, expressed in decimal, i.e. if Y=0.14%=0.14

where e.g. Y=Yield M, expressed in decimal, i.e. if Y= Yield M= 6.06%= 0.0606

spread-based, generalized

$$1.4cvn \quad V = VEX = \frac{(((2 * C)/(Y^3)) - (((2 * C)/(Y^3)) * ((1 + (Y/N))^{(-N * T)})) - ((C * T)/(Y^2)) * ((1 + (Y/N))^{((-N * T) - 1)}) - ((C/(Y^2)) * ((T + (T * (Y/N)))^{((-N * T) - 1)})) + (((1 + (C/Y)) * ((T^2) + (T/N)) * ((T + (T * (Y/N)))^{((-N * T) - 2)})))/10000}$$

where e.g. Y = Yield M, expressed in decimal, i.e. if Y = Yield M = 6.06% = 0.0606.

Duration (1.3), Convexity (1.4) Factorization, Change of Price to change of Yield:

$$1.10c \quad \text{generalized} \quad \Delta P = \Delta TAP = K * (CHY) + (0.5 * V * (CHY^2))$$

and where  $\Delta P = \Delta TAP = -\text{abs}(\text{Duration}1.3n) * (CHY) + (0.5 * (\text{Convexity}1.4cvn) * (CHY^2))$ .

Universal Duration, Convexity Factorization, change of Price for change of Yield:

$$1.11 \quad \Delta P = DP = -\text{abs}(\text{Duration}) * (CHY) + (0.5 * (\text{Convexity}) * (CHY^2)).$$

A Partial Differential Process and Algorithms for Change ( $\Delta$  or  $\delta$ ) in Price (P) with respect to Yield and Time, in Discrete Form, where  $\Delta P$  occurs over Points in Time, Figure 27:

Formula 1.111: 
$$\Delta P = A + B + C + D$$

where,

$\Delta P$  = change in bid price, for given changes in yield and time

$$A = -\text{abs}(\text{Duration}) \times \text{Price}(\text{dirty}) \times \Delta Y$$

$$B = \frac{1}{2} \times \text{Convexity} \times \text{Price}(\text{dirty}) \times (\Delta Y)^2$$

$$C = \text{Theta} \times \text{Price}(\text{dirty}) \times \Delta t$$

$$D = -(\Delta \text{ Accrued Interest, for given } \Delta t),$$

wherein,

Y (YTM), computed on applicable day-count basis (Formula S.1 or Formula S.2)

Duration and Convexity, standard modified annualized (Formulae S.3 or 1.3, and S.4 or 1.4)

Theta ( $\theta$ ) recalculated at cash flow dates, such a theta:  $\theta = 2 \ln(1+r/2)$ , wherein  $r = \text{ym}$

Price (dirty) equals bid price, plus accumulated interest (an accumulated cash receipt)

$\Delta t$  is the elapsed time between two points in time on which the estimations are made

$\Delta P$  rounded to nearest pricing gradient per market price convention,  $\Delta P$  occurring  $\Delta t$ .

Arbitrage differential is the difference of precise  $\Delta P$  and actual market price change.

The process can be used for a single security or group thereof, each single security processed separately, or as a set of securities in aggregate weighted summation, Figure 28:

Formula 1.112

$$\Delta P_p = A_p + B_p + C_p + D_p$$

where p is on a portfolio basis, each security having a portfolio coefficient based on its portion of the present value, with such Aggregate Value Calculations for Portfolio utilized, establishing the Aggregate Values for Portfolio, comprising the identified process variables.

Processing Spreadsheets for 2.1 and 2.2 are constructed at minimum with Columns, for each point in time at which the change in price or yield is to be determined by calculation, for each security, comprising the Rows of the spreadsheet, at each column row variable value:

columns Bid Price (P); Maturity (T); Coupon (C); Accrued Interest outstanding; and Yield (Y), where source of values per security are printed, quoted or digitalized information;

Duration; Convexity; and Theta columns follow, means for performing algorithms;

columns computing values of A, B, C and D between two time points per security;

column computing  $\Delta P$  from A, B, C and D; column rounding value to price gradient;

column posting actual change in price per security over the two points in time;

column calculating the arbitrage differential of  $\Delta P$  minus actual change in price.

In application of the process Formulae 1.111 or 1.112, the returned calculations for a change in price are in accord with the change in market price, pricing within one gradient, here, a 32<sup>nd</sup> referring to U.S. Treasuries. Because up to 1/64<sup>th</sup> of one percent is under- or over-valued relative to notching of market price, Arbitrage Differentials are identified and sorted. Figure 29 using S.1, S.3 and S.4; Figure 30, S.2, 1.3 and 1.4: Prices, *The Wall Street Journal*.

An apparatus, generating financial data, an analytic valuation engine, Figure 31 (and 32). Apparatus processes input values from a data-feed, stored memory or by simulation, for a security, or for securities in a portfolio, with respect to C, Y and T. The system calculates the governing yield, the Yield M, for the security or for portfolio, applying coded algorithms of Formulae 1, and sends the calculated value(s) on to the arbitrage engine, together with the security's market yield values determined using prior art Formula S.1 or S.2. The governing yield value and the market yield values are sent to further analytic processing, wherein using yield data, duration and convexity (and theta) data per Formulae 1.3, 1.4 (1.111), and wherein per market yield data, duration and convexity (and theta) data per Formulae S.3, S.4 (1.111). Sending the governing yield, and its convexity, duration and theta, data set to data storage, and computing the factorization per Formula 1.10 (1.111), whereas sending market yield data set to data storage and computing its factorization per Formula S.5 (or 1.111). Apparatus has means tabling, graphing and charting data, useful to analyzing pricing or setting hedge ratios.

An apparatus, processing data and transaction, comprising automated arbitrage engine. The apparatus processes input data from storage or data-stream of analytic valuation engine. The apparatus updates the market pricing of security and the security's market yield data from real-time data-feed. An arbitrage differential is computed between market yield and security's governing yield (precise price change vs. actual). Transaction costs adjust arbitrage differential. Arbitrage opportunities are sorted according to profit, spread or embedded premium. A relative value trade basis of  $\text{Yield M}^P - \text{YTM}^P$  data, generated Figure 20,  $\text{YTM}^P$  as S.1 or S.2. Apparatus rechecks pricing of opportunities and executes profitable transaction. Apparatus contains and updates inventory of securities, all transactions updated to storage. Apparatus provides to displaying opportunities, inventory and data to user. Figure 33 (and 34).



A unique type of financial security, a Replicated Equivalent Primary Security (REPS), wherein a REPS is engineered from available Primary Securities, e.g. fixed-income securities. REPS are unique composite securities, which replicate the targeted characteristics of an existing or non-existing primary security according to specified criteria. By replication, it is  
5 herein meant that a targeted primary security, such as a Treasury, Bond or Stock, is reproduced by an engineered manufacturing process from a set of similar category primary securities. For example, for a bond, the REPS is composed from other bonds of comparable credit risk and feature sets (i.e. coupon payment periodicity), so as to provide a critical identity of the target, such as cash receipts which match the target bond's, both in amount and in the  
10 timing of cash receipts. The target can be an analytical value, i.e. the duration of a portfolio.

As an example, assume a U.S. Treasury Note matures in exactly two years. A Treasury security in general is of the highest credit quality, which is equivalent in quality only to other Treasury securities. A holder of the Note will receive interest payments every six months until maturity, plus the redemption of principal to par on the final payment date. This cash receipt  
15 sequence is another key identity of Treasuries, along with the facts that Treasuries are devoid of optional features (early call or sinking fund provisions) and are priced according to unique determinants, such as proportioning annual time increments at actual days divided by 365.

Let this two-year to maturity Treasury carry an interest coupon of 6.00 percent, payable on a semi-annual basis, and let the owner hold a face amount of this bond equal to  
20 \$100 million. Thereby, in six months, the bond holder will receive \$3 million, in one year, the holder will receive another \$3 million, in one-and-a-half years, the holder will receive a third interest payment of \$3 million and at two years, the holder receives \$3 million in interest plus the redemption of principal at par of \$100 million.

Given that this example target instrument is a Treasury Note, it is possible to replicate the amount and timing of cash receipts, without alteration of the credit quality, through alternative Treasury instruments, assuming these latter Treasuries provide cash receipts about the dates payable by the target Treasury. It is possible to replicate the example target's cash receipts with either other coupon-bearing Treasury Notes or Bonds, or with zero-coupon Treasury STRIPS. The target criteria can be a REPS's duration, for immunization or hedging.

Method of manufacturing a replicated equivalent primary security comprises steps of:

identifying a target primary security, available in the market or of hypothetical design, or a target portfolio, by C, Y, T and P data, determining its cash receipt amounts and dates;

identifying such primary security, available in the market or of hypothetical design, maturing at or liquidated on date of a cash receipt from the target, one security for each date;

constructing a set of simultaneous equations, one for each cash receipt date of target, applying for each date each primary security's cash receipt, and summing each receipt at date;

solving set of equations simultaneously, returning coefficient per issue (basis of 100);

calculating for each security in set its face value, of its coefficient times target's face.

Method of manufacturing a replicated equivalent primary security further comprises:

identifying a target primary security, available in the market or of prospective design, or a target portfolio, by its C, Y, T data, determining its duration from C, Y, T, and price;

identifying each primary security by its immunization sensitivity of duration, which in aggregation with one or more such identified security, combining together to target duration;

identifying the duration data for the composite replicated equivalent primary security.

The Figure 35 diagrams the method of replication by targeting receipts and duration.

Cash receipt replicant method demonstrated, a 6.00% p.a. coupon, 2-year, Treasury Note is the target security, face amount of \$100, paying in six months, one year and one-and-a-half years, each of those dates coupon cash receipt of \$3; at 2 years, \$3 plus \$100 principal.

A replicated equivalent primary security of Treasury Notes or Bonds, matching target:

- 5        1)     T-Note, maturing in six months, carrying a 5.00% coupon;
- 2)     T-Note, maturing in one year, carrying a 5.50% coupon;
- 3)     T-Note, maturing in 1.5 years, carrying a 7.00% coupon;
- 4)     T-Note, maturing in 2.0 years, carrying a 4.50% coupon.

Setting simultaneous equations to replicate based on cash receipt amounts and dates:

10	Equation	1)	2)	3)	4)	=	T)
	a)	$1.025 \times 1)$	$+ 0.0275 \times 2)$	$+ 0.035 \times 3)$	$+ 0.0225 \times 4)$	=	3.00
	b)	$0 \times 1)$	$+ 1.0275 \times 2)$	$+ 0.035 \times 3)$	$+ 0.0225 \times 4)$	=	3.00
	c)	$0 \times 1)$	$+ 0 \times 2)$	$+ 1.035 \times 3)$	$+ 0.0225 \times 4)$	=	3.00
	d)	$0 \times 1)$	$+ 0 \times 2)$	$+ 0 \times 3)$	$+ 1.0225 \times 4)$	=	103.00.

15        Solving for these equations gives the following face amounts per security in replicant:

$$1) = \$0.672959 \quad 2) = \$0.689783 \quad 3) = \$0.706984 \quad 4) = \$100.733496.$$

A replicated equivalent primary security of Treasury Zero-Coupons, matching target:

- 1)     six-month zero-coupon U.S. Treasury STRIPS, face value \$3
- 20    2)     one-year zero-coupon U.S. Treasury STRIPS, face value \$3
- 3)     1.5-year zero-coupon U.S. Treasury STRIPS, face value \$3
- 4)     two-year zero-coupon U.S. Treasury STRIPS, face value \$103.

Alternate replicated equivalent primary securities for a target security or portfolio:

Target Security, a U.S. Treasury Note, held to mature 5/15/99, as of April 3, 1996, Figure 39:

	Maturity:	May 1999
	Coupon:	6.75% per annum, semi-annual payments
5	Prices: Bid/Ask	102:07; 102:07 / 102:09; 102:11
	Face Value:	\$50 million
	Best Price:	\$51,140,625
	Accrued Interest:	\$1,300,205
	Total Cost (P+AI):	\$52,440,830
10	Duration (S.3 mod. ann.):	2.782972.

Replicant A: Replicated Equivalent Primary Security having intermediate T-Notes, comprising Figure 36, to matching target primary security's cash receipts by amount and date.

15 Replicant B: Replicated Equivalent Primary Security having zero-coupon STRIPS, comprising Figure 37, to matching target primary security's cash receipts by amount and date.

Replicant C: Replicated Equivalent Primary Security having intermediate T-Notes, comprising Figure 38, to matching target primary security's cash receipts by amount and date.

20

The Figure 40 provides tabulation of the target and the assorted REPS, respecting the cost of buying or selling each one of them respectively. The Figure 41 provides tabulation of permutations of target versus REPS arbitrage opportunities and sorted arbitrage opportunities.

A replicated equivalent primary security (REPS) generator, comprising means to input a target security's data values of C, Y, T and P for target, and pricing sensitivity, for instance, duration. The target parameter to be matched by REPS is set, of cash receipts, or of pricing sensitivities. Possible REPS are engineered from available securities of same security typus, such manufacture operating by replicating method, such a method is the method of Figure 35. Generator having means sending alternate REPS to arbitrage engine, with price, variables and pricing sensitivities, with comparison of target. Generator having means displaying alternate REPS to monitor or rendering device, with comparison target. Generator with means to select a specific REPS to be manufactured, and to execute manufacture in transacting specific REPS. Generator having means to deliver and render REPS to end-user or internal memory, updating to storage, wherein having means to display all transactions at user request. The Figure 42.

An apparatus, processing data and transaction, comprising automated arbitrage engine, the Figure 43. The apparatus having means to input data from REPS Generator. The apparatus means updating the market pricing of alternate REPS and the target security's from real-time data-feed. An arbitrage differential is computed between target and each alternate REPS, and between pairings of each REPS, whereas a profitable arbitrage transaction comprises profit from selling either target or an alternate and buying either an alternate or target. Transaction costs adjust arbitrage differential. Arbitrage opportunities are sorted according to profit, spread or embedded premium. Apparatus rechecks pricing of opportunities and executes profitable transaction. Apparatus contains and updates inventory of securities, all transactions updated to storage. Apparatus provides to displaying opportunities, inventory and data to user.

An integrated computer-based financial information and transaction processing system providing for: the analytic processing; the manufacture and delivery of replicated equivalent primary securities; assessment of arbitrage spreads and execution of arbitrage transactions.

The system compose those business logic computational engines as three core server-based systems: the analytic valuation engine, with its methods, data, processes, means and functions described Figure 1 through Figure 32; the replicated equivalent primary security generator, with its methods, data, processes, means and functions detailed Figure 35 through Figure 42; and an automated arbitrage engine, with spread arbitrage differentials contained in Figure 20, with notching arbitrage differentials contained in Figures 30 and 31, and with replicant arbitrage differentials contained in Figures 40 and 41, and with its methods, data, processes, means and functions detailed therein and in Figure 33, Figure 34, and Figure 43.

Each of the component business logic servers receives market pricing data through a real-time financial data-feed, whereas the relevant signal data (i.e. for analytic valuation: of security typus, credit rating, C, T, P) are delivered as arrays for computational processing. The output data of the analytic valuation engine and REPS generator are sent into the arbitrage engine for processing, as well as to displays or external destinations necessary for functioning, manufacture, monitoring and control. Output from any of the engines can be sent to terminals and printers, and each of the engines is linked to storage medium allowing output and results to be stored to memory, so as to enable monitoring, review and assessment of the operating systems, trades, sales, inventory and P&L. Automated control sequences are established, to accomplish assembly of REPS, and execution of computer-driven transactions, effected over tele-communications, with exchanges, dealers, brokers or investment entities. The system is secured by encryption, and gate-keepers and firewalls between cores. Figure 44.

In addition to the cause for proprietary in-house investment management and analytic technologies, these adding profitability and security to in-house trading, investment and insurance portfolios, the market is screaming for superior fixed-income mutual funds. A low-risk, guaranteed profit, index out-performing, portfolio investment and management method.

5 Utilizing the U.S. Treasury instruments, being the lowest-risk, guaranteed return, credit risk-free, fixed-income investment, U.S. Treasury portfolio managers strive to enhance returns, but with the burden of increased risk, by taking on risk as lengthier expirations, yield curve weightings, speculative tactics and credit spread on U.S. agency paper. My independent study of the mutual fund industry dedicated to U.S. Treasury funds, shows that this sector (as  
10 the index performed) earns returns in excess of shortest-term U.S. Treasury paper. Yet unlike tactics above, my study showed that indexed aggregate sector performance approximated, and could be mirrored by a, randomly selected, roughly evenly distributed, ladder index portfolio. Allocating funds in to Treasury Notes or Bonds, spanning the short- to mid-term maturations.

The reward of the managed U.S. Treasury mutual fund sector can thus be replicated  
15 simply by a ladder allocation. Using allocation of a ladder portfolio, a low-risk foundation, achieving the average returns of professional portfolio managers, yet earning without chance.

Wherein, the centerpiece of the strategy is a ladder-based U.S. Treasury portfolio, comprised of coupon-bearing Notes and Bonds which span the short and medium terms. This composition itself can be expected to earn approximately, and with vastly less risk, the return  
20 of the mutual fund industry sector, and without its managerial overhead or speculative risks.

Moreover, by virtue of balanced capital over a maturity spectrum, the structure has a reliable yield-curve risk, as it tracks the developments in the yield curve, plus it provides a higher return to variance, a natural awareness of risk-points and a reduced reinvestment risk.

The business logic allows the ladder to run, with sequentially, each issue maturing. The logic need not allocate in the shortest maturities, since, soon after start-up, the portfolio de facto maintains the shortest maturities, as the cycle brings to maturity yesterday's Note, whereas at identical maturation (expiration), Treasury Bills, Notes and Bonds convergence.

5        Such management of such portfolios is easy, little is required, just an automated reinvestment of matured funds to the end of the portfolio line. Skilled management can add, that is also because the character of such ladder mutual funds can be so readily approached. The intricacies of optimization, of loss avoidance, profit capture and of continuous perfection of the investment composition are simple, for example, the coupon interest payments received  
10       can be accumulated and reinvested periodically, and while accumulating on cash account, providing operational liquidity, i.e. to periodic cash-outs of investors and administrative costs.

      This rolling ladder portfolio mutual fund's first enhancement is discussed above - the arbitrage to mutual fund sector performance accomplished by the reduction of assumed risk. The final two enhancements are similarly derivative-free arbitrages. The second enhancement  
15       to return is the group of relative-value managements to the exact portfolio holdings over time.

      Remembering that any coupon-bearing Note or Bond issue is itself a fixed ladder portfolio of cash receipts, and that such cash receipts are zero STRIP equivalents, the business method identifies replications, the compositions and permutations of issues and their parts, which, by automated computerization, pricing out arbitrages on cash receipts, represented or  
20       divisible within portfolio holdings, against replications by available instruments not in hand.

      The final arbitrage is risk-free basis, capturing the reversional pricing dynamic between a ladder portfolio, or its single issues, and relative, governing yield, valuations. The engines and system can implement, allocate and transact the fund, by this method, Figure 45.



A process for transforming data, such data in nominal or monetary units, to ameliorate the influence of inflation and to transform the data mathematically to revealing distribution. The financial data, of a value expressed as an amount of a currency unit, and/or its underlying object of value, are discrete samplings of a variable taken over time. The process utilizes the actual monetary units of a data set, wherein such units include the group of world currencies, and provides the data array of the actual nominal value of a security or an underlying variable, spanning a range of time, with discrete sample values identified within the given time-frame. The sequential array of a security's price (value) data over time, or of a variable with nominal valuation, such as the temporal development of insured losses due to catastrophes or defaults.

The data are processed to spread-sheets, rendered in charts, or statistically described, in actual nominal data values over a span of time. For example, Figure 46, represents insured U.S. actual losses, in nominal U.S. dollars (in billions), for the financial variables of deposits closed and net deposit loss (closed plus recovery) relating to loss amounts of the bank system, and of insured catastrophe and statutory underwriting loss relating to property & casualty loss.

The Figure 47, left side, provides the data array of each said variable's nominal value data.

Next, the data value at a specific date, if said date is post-1972, the nominal value is multiplied by its corresponding inflation-adjusted scalar, whereby said scalars are determined by setting the year 1972 at 1.00, unity value. A Federal CPI number for the currency country comprise source data that can be utilized and processed to achieve the scalar for each year, herein, the U.S. Consumer Price Index (CPI). Proceeding in order, for each year, its CPI inflation is subtracted out from the proceeding year's rolling (decreasing cumulative inflation) total, such a scalar for 1973 is: 1972 scalar - 1973 inflation rate, i.e.:  $1.00 - 0.0625 = 0.9375$ . The scalar for 1974 is the 1973 scalar minus 1974 inflation, each year proceeding forward.

The domestic U.S. dollar scalars for the years, 1972 through 1996, are presented in Figure 47, and, right side, the 1972-adjusted U.S. dollar values for the variables are arrayed. The Figure 49 is graphical representation of 1972-adjusted data values for the four variables.

Within the context of most financial formulae describing the pricing relationship with  
5 respect to time is the assumption that the financial variable is independent and has lognormal distribution. The statistical evidence of some financial variables or securities supports this. It is appropriate and often necessary, in the implementation of the financial formulae, to utilize the natural logarithmic values of the variable or security. Hence, the next part of the process transforms the 1972-adjusted data values into logarithmic values, in Figure 48, left side. As  
10 the final part of the transformation, the change (“delta”) in the logarithmic data from one time point to the next, in the variables herein utilized, from one year to the next, is computed, and is displayed Figure 48, right. The processed log and delta data are graphically represented in the Figures 50 and 51, respectively. Correlation between these variables is visually apparent.

The data values of each variable or security can be separately processed or processed  
15 in tandem with its transformed values, in addition to being processed and graphed within a collection of variables that are evaluated. The Figure 52 graphs the log values of the 1972-adjusted data for deposits closed, in tandem with its log delta values. The Figure 53 graphs the log values of the 1972-adjusted data of deposits closed in comparison with deposit losses. The Figure 54 graphs the log delta 1972-adjusted data for deposits closed and for deposit losses.  
20 The Figure 55 graphs the actual nominal values of net statutory underwriting and catastrophe losses for the Property & Casualty (P&C) industry on a consolidated basis. The Figure 56 graphically represents the P&C industry’s 1972-adjusted dollar amounts of statutory and catastrophe losses. The Figure 57 charts the log values of the 1972-adjusted data for statutory

and catastrophe losses, whereas the Figure 58 charts the log delta 1972-adjusted data. The Figure 59 presents the actual nominal underwriting loss and the loss from catastrophes, revealing the contribution of the later to the overall performance of the P&C underwriting. The Figure 60 graphically represents the log and delta log values for catastrophe losses, and the Figure 61 charts the log and delta log values for statutory and catastrophe losses. By charting, the trends of the cleaned data variables are visually apparent.

To further provide information on the characteristics of the financial variables or securities, descriptive statistics of the variables are processed and assembled. The Figure 62 comprises the descriptive statistics for the financial variable, deposits closed, over the modern (post-1972) period, for each of the four data sets, of actual nominal, inflation-adjusted, log of the adjusted, and the change (delta) of the log. The Figure 63 contains descriptive statistics for the variable, catastrophe loss, over the modern period, for the four sets of processed data. By processing the descriptive statistics, the key statistical values of the variables are rendered, which can be further utilized in financial formulae requiring such values, i.e. for the mean.

For comparison purposes, as would be appropriate for assessing two financial variables which are involved in a swap, the descriptive statistics of each variable, over identical time periods and on identical processed value basis, are rendered side-by-side. Such is provided in Figure 64, wherein the descriptive statistics of the log adjusted values of deposits closed and catastrophe loss for the period 1979-1995 are represented. The Figure 65 comprises the log delta values for those two variables over the length of their entire history. By doing so, it becomes apparent that, over the entire history, the change in log values are nearly identical for these two variables, whereas their standard deviation and variance differ.

The distribution of the variables' respective processed values are visually assessed by means providing histograms of the values. For many financial formulae, the assumption of log normal distribution is required to implement them within their appropriate defined construct. Utilizing histograms of the non-log values, the log normal distribution appears as a sharp rise, with a long tail, and for processed log data values, the standard bell shape should be evident. The Figures 66, 67 and 68 contain histograms for the financial variables, over defined time-periods, on actual nominal, log-adjusted, and delta log, processed bases. As is evident, the actual nominal values of deposits closed and catastrophe losses spike and tail roughly as might be expected from log normal variables, and their delta log histograms roughly match the bell distribution, caveat being the limited sample size involved. Such information is vital in assessing the efficacy of valuation and expectation methodologies assuming log normalcy.

As further means for processing data sets of financial variables or values of securities, the data sets are adjusted by a relative growth scalar appropriate for the variable or security. For example, for the deposits closed, insured by the FDIC, an appropriate growth scalar is the rise in the insured deposit base as a percentage of the total deposit base since the inception of the variable, (insured) deposits closed. Figure 69 contains the data array of the percentage of deposits insured respective total deposits for each year (data point). For each year, the percent is divided by the first point's (the year 1934 in this case) percent, to render the year's scalar. The actual nominal value and the 1972-adjusted value for each year are divided by the scalar for that year, with the log and delta log values then calculated from the growth-scaled values. On this basis, the portion of the rise in deposits closed attributable to increase of the insured deposits relative to the total deposit base is removed, resulting in clean, growth-adjusted data. Descriptive statistics are processed for the growth-adjusted data array, as in the Figure 70.

As a second example of growth-adjustment data array processing, the nominal values for insured catastrophe losses are scaled by the growth in the insured property base, respective an original point in time. Utilizing the fact that the insured property base increased by 80% in the period 1988 to 1995, the growth over that period commencing in 1988 is attributed, here in linear additive fashion, such that for 1988 the scalar is 1 and for 1995 the scalar is 1.8. The scalar for each year is applied as a denominator to the year's catastrophe losses, Figure 71.

Histograms for the growth-adjusted data arrays are processed, contained in Figure 72. The import of growth-adjustment is visually evident in the histograms - the resulting histograms are broader across bins, or more normal with respect to their prior distributions. The broadened, frequency rich probability distribution is desirable with respect to insurance, and reinsurance, especially to constructing robust models for multiple excess of loss layers.

A process to evaluate the incidence and likelihood of default or loss within the insured depository banking industry, comprising two separable and distinct processing methodologies. The first process creates two risk variables for deposit insurance and credit default analysis, the insured [Deposits Closed/Total Deposits] and the insured [Closed Assets/Total Assets]. The data array for each numerator and denominator are compiled, contained in the Figure 73. Utilizing those data arrays, the quotients (the variables) are processed as arrays, Figure 74, represented as  $\#/\#$ . The quotients are further processed as natural logarithms, expressed  $e^{(\#/\#)}$ . Utilizing either of the processed data arrays, the incidence of variable default is established, and are further processed into descriptive statistics, histograms and graphical representation. The Figure 75 presents the graph of the financial variable, [Deposits Closed/Total Deposits], said variable comprising a martingale probability of default frequency and relative magnitude. Figure 76, descriptive statistics and histograms of log of the risk variables and their log deltas.

The second process, useful for the evaluation of default likelihood on an industry basis or on an individual institution basis, compiles the data values for the variables, total deposits (TD), interest-bearing deposits (IBD), for total assets (TA), interest-bearing assets (IBA) and interest-bearing liabilities (IBL), wherein total assets equals total liabilities (TL), Figure 77.

- 5 Next, a variety of operating ratios (variables) are processed from TD, IBD, TA, IBA and IBL, including the ratios:  $TD/TA$ ;  $IBD/IBA$ ;  $IBD/IBL$ ;  $TD/IBA$ ;  $TD/IBL$ ; and  $IBA/IBL$ , Figure 78.

Once processed, these ratios can be utilized in combination to reveal the conditions effecting the safety of operations. For example, the Figure 79 graphically represents the ratios of  $TD/TA$ ,  $IBD/IBA$ ,  $IBD/IBL$  and  $TD/IBA$  on an banking industry-wide basis since 1934.

- 10 Evident from the data over time is that the wide safe operating margins, between the ratios  $TD/IBA$  and  $IBD/IBA$  constricted substantially, indicating an increasingly slim safety margin. The Figure 80, graphically representing the ratios  $TD/IBA$ ,  $TD/IBL$  and  $IBA/IBL$ , shows that the underlying relationship between  $TD/IBL$  and  $IBA/IBL$  changed beginning in early 1970s, such that whereas earlier the former was greater than the latter, their relation then inverted.
- 15 This key relationship is the primary cause of increased default probability, and when graphed together with the default risk variables of [Deposits Closed/Total Deposits] and [Closed Assets/Total Assets], it is transparent that the rise in default corresponds thereto, Figure 81.

The Figure 82 diagrams the process of cleaning and preparing nominally valued data: taking actual nominal value data in array; adjusting nominal data by inflation scalar, and/or by growth scalar, as array; converting inflation adjusted data to natural log values as array; computing the log of the change (delta) between data points; rendering in tables, the nominal, adjusted, log, and log delta, data arrays; graphing in charts, the nominal, adjusted, log, and log delta, data arrays; computing and assembling statistics and histograms of the four data arrays.

A method and process of a technology for small samples create numerical data and statistical tools benchmarking the character of distribution functions in small sample environments, affording means for evaluation of the nature of financial data with respect to establishing its underlying distribution character.

5

The method and process of small sample technology is useful, simulating and demonstrating how a uniform random variable with a known distribution function occurs in small sample environments, i.e. in the absence of the large data sets by which the Law of Large Numbers is applicable in shaping frequency.

10

The method and process of the small sample technology can be used for any set of discrete data, and is useful for data sets which are time-series or serially correlated. Examples of small sample financial data are the annual aggregates of U.S. insured depository default and catastrophic property casualty losses.

15

The method and process of the small sample technology are especially designed for evaluation of data which may be a standard normal variable. Financial pricing formulas and theories imply, infer or require that the financial data is a standard or log normal variable. It can be applied to other disciplines' data sets.

20

The method and process of the small sample (typically  $N \leq 30$ , can be greater) technology comprises steps of:

A) randomly generating sequences of independent uniform random variables as separate data arrays, over the values of zero to one, each sequence separately seeded, each using a different seed clock rate;

- 5 B) taking the data arrays in pairs and performing a Box-Muller transformation on the two data arrays of each pair, using the formulas, one for each array, to generate numeric output of standard normal random variables:

Box-Muller: Std. Normal Random Variable  $V1 = \text{SQRT}(-2 \times \text{LN}(U(Ia))) \times \text{COS}(2 \times \text{PI} \times (U(Ib)))$

Box-Muller: Std. Normal Random Variable  $V2 = \text{SQRT}(-2 \times \text{LN}(U(Ia))) \times \text{SIN}(2 \times \text{PI} \times (U(Ib)))$ ,

- 10 where the Ia and Ib are from each of the two distinct, paired, data arrays;

C) utilizing the numeric output of a transformed data array, by taking small sample of its data to generating descriptive statistics of the small samples, which can then be used to indicate the probable and occurring statistical characteristics of standard normal variables

- 15 subjected to small sample environments, which further comprises:

- a) generating the mean, standard deviation, and sample variance, of the sample;
- b) generating a histogram for the sample frequency respective range of bin values;
- c) generating descriptive statistics for various sized samples of each data array;

- 20 D) utilizing the statistical descriptive characteristics of the small samples generated to evaluate and serve to benchmarking the nature of the distribution underlying financial or other data in small sample sets.



The method and process of the small sample technology further comprises alternate lognormal technology:

- A) taking the natural log of the transformed data arrays, this done after the step B) above;
- B) utilizing log numeric output to generating statistics and evaluating data distributions.

5

A method and process of small sample technology which comprises an alternate transform technology, creating a random variable without reference to more than one array:

- A) taking the data arrays in singleton and using formula for each array, generating output:

$$\text{Random Variable} = \text{SQRT}(-2 \times \text{LN}(U(Ia))) \times \text{COS}(2 \times \text{PI} \times (U(Ib))),$$

10

where the Ia and Ib are from the same sequence data array;

- B) utilizing a series formalism to relate the Ia and the Ib, eg.  $U_i$  and  $U_{i+1}$  respectively.

This latter type of generated random variable tests in large numbers as standard normal with respect to mean (of zero), standard deviation (of one) and variance (of one), irrespective.

15

The Figure 83 contains 10 random generated sequences of independent uniform random variables as separate data arrays. The Figure 84 contains the 10 data arrays in pairs, pairs A to E, after performing the Box-Muller transformation on the two data arrays of each pair, V1 and V2. Figure 85 contains descriptive statistics and histograms for each transformed data array, N=7. Figure 86 contains descriptive statistics and histograms for each transformed data array, N=15. Figure 87 contains descriptive statistics and histograms for each transformed data array, N=23. Figure 88 and Figure 89 contain descriptive statistics for each array, where N=48 and N=62. Figure 90, renders ten standard normal random variables per alternate transform, U of Figure 83.

20

Theta Modeling Technology with Mathematical Functions and Numerical Techniques  
 respective an Underlying State Variable, Theta, wherein, an investment or derivative security  
 is modeled by using a single underlying state variable, such a theta variable, insured deposits  
 closed. These have a positive nominal value, an actual amount of cash value. This underlying  
 5 state variable, theta,  $\theta$ , is held to follow an independent Markov process:  $d\theta/\theta = m dt + s dz$ .

This asserts that the future value of  $\theta$  depends on the known present values under a  
 continuous pricing constraint. As Wiener process,  $dz$  is related to  $dt$ :  $\Delta z = \epsilon \sqrt{\Delta t}$ .

Such theta variable depends solely on itself and time to define its expected drift and  
 volatility, which it redefines through the course of its life. Thus,  $d\theta/\theta = m(\theta, t) dt + s(\theta, t) dz$ .

10 For methods drawing from standard normal distribution, the log of the change of theta  
 over time and/or the log of theta at exercise should have this distribution. The theta approach  
 is useful where a target variable is not the price of a traded security, but it is useful there, too.

To creating a tradable instrument for a theta variable, assigning the function,  $f$ , as the  
 price of a security dependent on  $\theta$  and time. For instance, for the insured banking's variable,  
 15 the "deposits closed" and "deposit loss" are candidates for industry's theta. For the insured  
 catastrophe dollar risk, they are the "catastrophe loss" and "net statutory underwriting loss".  
 Variables are created from divers theta, such  $\theta_i$ , e.g.  $\theta_b$  and  $\theta_c$ , correlating respective losses.

For instance, for  $\theta$ (banking: of insured deposits closed or net deposit losses) and  $\theta$   
 (cat: of insured catastrophe or net statutory underwriting losses), let  $f(b)$  and  $f(c)$  be the  
 20 respective price of a derivative security with payoff equal to a functional mapping of  $\theta_b$  and  
 $\theta_c$  into the future. Let the processes of  $f(b)$  and  $f(c)$  be defined via Ito's lemma, where

$$df/f = \mu dt + \sigma dz. \text{ This stands for any } f(\theta).$$

On a continuous time basis, the change in the price of security (f) dependent on the banking losses is  $df(b) = \mu_b f_b dt + \sigma_b f_b dz$ ; (and for  $\theta_c$ :  $df(c) = \mu_c f_c dt + \sigma_c f_c dz$ ). An instantaneously riskless portfolio can be created from a combination of related  $f(b_i)$ , such that  $(\mu_1 - r)/\sigma_1 = (\mu_2 - r)/\sigma_2 = \lambda$ , where  $r$  = the present spot, risk-free interest rate at time 1 and 2.

5 Thus, for any  $f$ , being the price of a security dependent on only  $\theta$  and time, with

$df = \mu f dt + \sigma f dz$ , there is the parameter lambda,  $\lambda = (\mu - r)/\sigma$ , which is dependent on  $\theta$  and time, but not on the security  $f$ , estimating the market pricing of risk of  $\theta$  by stochastics.

The theta variable's  $\mu$  is the expected return from  $f$ . The expected drift,  $\mu$ , equals  $\mu f$ . Sigma,  $\sigma f$ , is the volatility of  $f(\theta)$ , and either positively ( $df/d\theta > 0$ ) or negatively correlates to  $\theta$ .

10 If negatively correlated, volatility =  $-\sigma$ , and  $df = \mu dt + (-\sigma) f (-dz)$ . Variance is  $[(\sigma^2)(f^2)]$  and  $dz$  is over an independent interval,  $dt = (T-t)$ . Using Ito's lemma, the parameter,  $\mu$ , relating  $\mu$  and the pricing function, is set as  $\mu^* f = df/dt + m \theta df/d\theta + \frac{1}{2} s^2 \theta^2 d^2f/d\theta^2$ .

The parameter Sigma is set,  $\sigma^* f = s \theta df/d\theta$ . This results in a differential structure  $df/dt + \theta df/d\theta (m - \lambda s) + \frac{1}{2} s^2 \theta^2 d^2f/d\theta^2 = r^* f$ .

15 This equation can be solved by setting the drift of  $\theta$  equal to  $(m - \lambda s)$ , and discounting expected payoffs at  $r$ , the present spot risk-free (usu. U.S. Treasury) interest rate.

Thus, under risk-neutral valuation, the drift of  $\theta$  is reduced from  $m$ , to  $(m - \lambda s)$ .

To constructing a valuation lattice, such in discrete time, introducing the notions of

delta,  $\delta = e^{(\sigma^* \sqrt{\Delta t})}$ , and of  $\mu = [2 * e^{(r^* \Delta t)}] / [\delta + \delta^{-1}]$ .

20 Hence,  $\sigma = \ln(\delta) / (\sqrt{\Delta t})$ . Next, setting values for  $\Delta t$ , sigma,  $r(t)$  and  $\theta$  (where,  $\theta = S$ , if modeling an equity security), calculating nodes of  $\theta$  at  $\theta(tk) = [\mu^k] * [\delta^{w(k(w))}] * [\theta_0]$ .

Substituting theta ( $\theta$ ) for a security (S), S isomorphic to  $\theta$ , affording an underlying random walk of  $w(k(w))$ , such that if  $w=(-1,-1,1\dots)$ ,  $\theta(t(3)) = [\mu^3] * [\delta^{(-1)}] * [\theta_0]$ .

In lognormal world, this relates:  $\ln \theta(tn) = [n * \ln \mu] + [w(n(w)) * \ln \delta] + [\ln \theta_0]$ .

This results in the equalities:  $\ln \delta = \sigma * (\text{sqrt } \Delta t)$  and  $dt = T/n$ . Substituting and letting  $k = n$ ,

5 such that  $tn = T$ , forms:  $\ln \theta(tk) = [n * \ln \mu] + [w(n(w)) * (\sigma * \text{sqrt } \Delta t)] / [\text{sqrt } n] + [\ln \theta_0]$ .

More simply, the expected value of  $\theta$ ,  $E(\ln \theta) = \ln \mu + \ln \theta_0$ .

The variance of  $\theta$ ,  $\text{Var}(\ln \theta) = (\ln \delta)^2$ .

The volatility of  $\theta$ ,  $\text{Vol}(\ln \theta) = (\ln \delta) / \text{sqrt } \Delta t$ .

For a pathing tree, the node value mechanic,  $\theta(tn) = [\mu^n] * [\delta^{w(n(w))}] * [\theta_0]$ , using  
10 logarithmic transform, node mechanic,  $\ln \theta(tn) = [n * \ln \mu] + [w(n(w)) * (\ln \delta)] + [\ln \theta_0]$ .

By the Central Limit Theorem, the term,  $w(n(w)) / (\text{sqrt } n)$  exhibits strong convergence to the standard normal distribution,  $N(0,1)$ . The term  $[n * \ln \mu]$  shows weak convergence to  $[(r - \frac{1}{2} \sigma^2) * T]$ , hence its robust implementation is limited to the rigors of discrete methods.

The term  $[\ln \theta(T)]$  is distributed as  $[(r - \frac{1}{2} \sigma^2) * T] + [N * \sigma * (\text{sqrt } dt)] + [\ln \theta_0]$ .

15 Non-log,  $[\theta(T)]$  is distributed as:  $[\theta_0 * e^{((r - \frac{1}{2} \sigma^2) * T)} + (N * \sigma * (\text{sqrt } \Delta t))]$ .

Valuation of a derivative security (S) based upon the state variable theta, example, the European call option, with realizable cashflow only at T, value today of P, with the functional mapping,  $f(\theta_0) = \max [\theta(T) - K, 0]$ , where K is strike price and  $\theta$  is held substitutable by S. By weak convergence, today's value,  $P(\theta)$ , based on  $\theta$  at T, derived over normal distribution:

20  $P(\theta) = [\theta_0 * N\{(rT + \ln(\theta_0/K)) / (\sigma * (\text{sqrt } dt)) + (1/2 \sigma^2 * (\text{sqrt } dt))\}] -$

$[K e^{(-rT)} * N\{(rT + \ln(\theta_0/K)) / (\sigma * (\text{sqrt } dt)) - (1/2 \sigma^2 * (\text{sqrt } dt))\}].$

$P = e^{(-rT)} * E(m)[\theta(T) - K]$ , where  $E(m)$  = expected value under risk-neutral conditions.

For any function,  $f$ , valuing a derivative security based on  $\theta$  that pays off  $f(T)$  at time  $T$ , the expected risk-neutral value is  $f = e^{(-rT)} * E(rn)(f(T))$ . This requires setting the growth rate of the underlying  $\theta$  variable in relation to  $[m - \lambda * \sigma]$ , rather than as  $m$  alone.

Thus, risk-neutral valuation for today's value,  $P(\theta)$ ,  $P(\theta)=f(\theta)$ , of a derivative security  
5 paying off  $f(T)$  at time  $T$ , is equivalent to the risk-free discount over period  $(0,T)$  of its expected risk-neutral future pay-out. This narrow evaluation is valid for  $f$  only over the continuous segment  $(0,T)$ , with determinable values of  $F(0)$  and  $F(T)$ . Lattices which subdivide this segment, are weakened, if their  $\Delta t$ -parameters, i.e.  $\Delta t = (T-t)$  with  $0 < t < T$ , are modeled using analytic values from  $(0,T)$  data sets. Any methodology which relies on  
10 convergence to a normal distribution for its valuation, for instance, or a sampling therefrom, is strictly consistent only for European-style derivatives, that is, having exercise only at  $T$ , but not continuously throughout the segment  $(0,T)$ . Also, it assumes the security can gain or lose value during  $(0,T)$ , with the value of the security always non-negative. The payoffs of the  $\theta_b$  and  $\theta_c$  securities can be European, if these stem from the single terminal condition of  $\theta$  at  
15  $T$ : the selected  $\theta$  variables are annual aggregates, they begin each year at  $\theta=0$  and end the year at  $T$ ,  $\theta \geq 0$ , European  $(0,T)$  events. For rigorous risk-neutral valuation, strict conformity can only be assigned under a European-style  $(0,T)$  segment, variable and  $\theta$ -based security. Such a  $\theta$  can substitute for a continuously traded security after adjusting for the conditions that  $\theta$  at  $T = \sum \theta_i$ , each  $\theta_i$  occurring and aggregating discretely over  $(0,T)$ . For continuous  
20 trading, full data along the annual path of such  $\theta$  over  $(0,T)$  are required to be available.

A valuation function,  $V$ , for a security or derivative dependent only on  $\theta$  and time:

$V(t,0)=V(t)= V_0 * e^{\{(r - \frac{1}{2} \sigma^2)t + (N * \sigma * (\sqrt{t}))\}}$ , where  $V(0,T)$  is identity of  $\theta(0,T)$ .

To constructing a swap, e.g. between the insured deposit losses and catastrophe losses, modeled on  $\theta(b)$  and  $\theta(c)$  respectively, each having valuation function,  $f(b)$  and  $f(c)$  respectively. The value of the swap to the payer of the deposit losses,  $f(b)$ , assuming the swap of all year-end aggregate losses:

$$V = e^{-rT} * \{E(rn)[f(c)(T) - f(b)(T)]\}, \text{ where } E(rn) = \text{risk-neutral Expectation.}$$

Though a swap is composed of two sides, its value,  $V$ , is a single function. Thus,  $V$  is a single derivative instrument modeled by the expectation of the two functions, each respective of its own single  $\theta$  variable. Consequently, this function,  $V$ , models a security dependent on unrelated underlying variables,  $\theta(i)$ . Each  $\theta(i)$  follows a stochastic process of form:  $d\theta/\theta = \mu_i dt + \sigma_i dz_i$ , with  $\mu_i$  and  $\sigma_i$  the expected growth and volatility rates, the  $dz_i$  being Weiner processes, then substituting  $V$  for  $f$ , the total loss swap,  $V$ , has the form:

$$dV/V = \mu dt + \sum[\sigma_i dz_i], \text{ with } \mu \text{ being the expected return of the swap.}$$

Component risk of the return to the  $\theta(i)$ ,  $\sum[\sigma_i dz_i]$  are adjusted if  $\theta(i)$  are correlated.

Brownian motion defines the change in the value of a variable as related to the variable's initial value and characteristic deviation, as well as to distinct random perturbations resonating variance over independent intervals. It is a discrete process that approaches continuous form when the intervals are small and uncorrelated. Pinned simulation fixes an initial and terminal value for the variable, then developing the value path in between.

An expression of  $\theta$  with respect to time and to Brownian motion, the life of  $\theta$  over  $(0, T)$  and projected Brownian motion in simulation, can be related in derivational form:

$$\theta(t, B) = \theta_0 * e^{[(r - \frac{1}{2} \sigma^2)t + \sigma B]}.$$

This above equation values without preference to risk, obtaining risk-neutral results.

For geometric Brownian motion, the theta variable must be lognormal in functionality (i.e. its natural log values must show distribution in line with a standard normal population).

To implementing this when modeling theta of such distribution, as respective of time only:

$$\theta(t) = \theta_0 * e^{[(r - \frac{1}{2} \sigma^2)t + \sigma \sqrt{t}]}.$$

5 The weak convergence by  $\Delta\theta(t)$ , requires only that the natural log of the change in theta shows a normalized distribution and characteristic variance (not necessarily  $\sigma^2=1$ ).

Allowing the notation,  $\theta(t,B) = \theta_0 * e^{[(r - \frac{1}{2} \sigma^2)t + \sigma B]}$ , the change in  $d\theta$ , measured at the terminal values  $(0,T)$ , with  $t=T-0$ , can be derived as:  $d\theta = \theta(B)dB + [\theta(t) + \frac{1}{2} \theta(BB)]dt$ ;

its partial derivative input parameters:

10 
$$\theta(B) = d/dB \text{ of } \theta(t,B) = \sigma * \theta;$$

$$\theta(BB) = \sigma^2 * \theta; \text{ and } \theta(t) = [r - \frac{1}{2} \sigma^2] * \theta.$$

This computes as  $d\theta = \sigma * \theta * dB + [(r - \frac{1}{2} \sigma^2)\theta + (\frac{1}{2} \sigma^2)\theta]dt$ , and can be reduced to:

$$d\theta = \sigma * \theta * dB + r * \theta * dt.$$

For normal theta variables, respective only to time and theta:

15 
$$\theta(t) = \theta_0 * e^{[(r - \frac{1}{2} \sigma^2)t + (N * \sigma * (\sqrt{t}))]; \text{ and}}$$

$$V(0,T) \text{ as the identity of } \theta(0,T), V(t,0)=V(t)= V_0 * e^{[(r - \frac{1}{2} \sigma^2)t + (N * \sigma * (\sqrt{t}))]}.$$

This requires only that the natural log of the change in theta, hence, in  $V$ , has a characteristic, normal distribution.  $N$  represents a sampling off the standard normal distribution, e.g.  $N=\epsilon=\phi$ .

Monte Carlo simulation is a discrete methodology that is based on the Law of Large  
20 Numbers (e.g. in large numbers of sampling sequences). The life of the security is subdivided into  $n$  intervals, each of length  $\Delta t$ . Using  $s \equiv$  volatility, and  $m \equiv$  risk-neutral growth rate, of  $\theta$ :

$$\Delta\theta = m * \theta * \Delta t + s * \theta * \epsilon(\sqrt{\Delta t}), \text{ where each simulation run has } n \text{ drawings, one per } \Delta t.$$

For a multiple state  $\theta$ :  $\Delta\theta_i = m_i \cdot \theta_i \Delta t + s_i \cdot \theta_i \cdot \epsilon_i(\sqrt{\Delta t})$ , with  $\theta_i$ : ( $1 \leq i \leq n$ ). If the  $\theta_i$  are correlated, implement correlation between the  $\theta_i$ ,  $p_{ik}$ , and also between the  $\epsilon_i$  and  $p_{ik}$ .

The Figure 91 diagrams the mathematical programming functions for theta variable.  
The Figure 92 diagrams the mathematical programming functions for security based on theta.

5 For further embodiment of theta, and as respective reinsurance and actuarial sciences, see inventor's published work, "Forecasting Expectations of Insured Depository Default and Catastrophic Losses". The publication further elaborates the purposes and mechanics of theta.

Regarding the use of a put-option model for deposit guarantees, i.e. insurance against deposit losses, a deposit insurance guarantee is not isomorphic to an European put option, as  
10 any recovery of underwritten (collateralizing) assets, occurs later than the deposit loss occurs. Asset recovery or loss development does not occur simultaneously to default or catastrophe.

An OAS/martingale lattice, featuring the development of recovery or loss over time, implemented in diagram schematic, Figure 93, an Option Adjusted Spread style lattice. Having an initial node, at  $t = 0$ , at which node having value at given  $t$ , expressed as  $e(0) = 1$ .  
15 From initial node, binary pathing, to the upper node at time  $t = 1$ , processing  $1 - M(0)h(1)$ , wherein  $M$  is martingale for the given  $h$ , and wherein  $h$  is distinct time interval, value at upper node of  $e(1) = 1$ , a state wherein no default or loss occurs; to the lower node at time  $t = 1$ , processing  $M(0)h(1)$ , value at lower node of  $e(1) = E(L)$ , a state where default or loss occurs, wherein  $E(L)$  is the risk-neutral expectation of  $L$  after asset recovery or loss development.

20 From the upper node at time  $t = 1$ , binary pathing: to an upper node at time  $t = 2$ , processing  $1 - M(1)h(2)$ , value at upper node of  $e(2) = 1$ , wherein no default or loss occurs; to a lower node at time  $t = 2$ , processing  $M(1)h(2)$ , value at lower node of  $e(2) = E(L)$ , a state where default or loss occurs. If lower node occurs, follow path relation of lower node at  $t = 1$ .



From the lower node at time  $t = 1$ , binary pathing: to an upper node at time  $t = 2$ , processing  $1 - R(1)h(2)$ , wherein  $R$  is conditional probability of recovery or development for given  $h$ , a value of  $e(2) = E(L') - \text{Expenses}$ , a state wherein recovery or development occurs; to a lower node at time  $t = 2$ , processing  $R(1)h(2)$ , value at lower node of  $e(2) = L$ , wherein  $L$  is the actualized settled value, a state wherein asset recovery or loss development is realized.

The importance of new and useful valuation technologies, such as the above lattice, for the field of insurance, whether for depository banking, catastrophe insurance or other line, as well as new sources of reinsurance industry capital, and new transactions to insure public interests, are made manifest in inventor's published work, "Forecasting Expectations..."

A financial swap transaction, comprising a reciprocal netting transaction, functioning as reciprocal treaty reinsurance between depository banks and Property & Casualty insurers, wherein such swap transaction the banks reinsure insurers against catastrophe losses, paying for such losses as specified under said reciprocal treaty, and wherein the insurers reinsure banks against depository default, paying for such loss amounts as specified under said treaty.

Such swap transaction is organized between the FDIC (representing depository banks) and the NAIC (representing P&C insurers), wherein some public funds of the Bank Insurance Fund (BIF), or of the BIF + SAIF (Savings Association Insurance Fund), fund as surplus capital.

Such surplus capital pays the applicable cat loss to insurance industry holders of a cat loss security. Said security pays its holders, such as NAIC members and P&C cat insurers. In turn, said holders pay, through their treaty organizer institution, NAIC, applicable deposit loss to the surplus capital fund managed by the FDIC and the NAIC. Such payment by cat holders, the deposit guarantee, is activated when the FDIC closes an FDIC insured depository bank. The swap enables reciprocal netting at the FDIC/NAIC administered capital. The Figure 94.

A calculator, portable and pocket-sized, having means for performing computations and for solving equations, including coded resident functions of the financial engineer. The calculator affords every financial professional, whether a financial analyst, actuary, risk or credit manager, in the derivative, bond, stock, mortgage or insurance industry, the requisite  
5 hardware environment, housing functional numeric processes, coded applications and reference resource items tailored to the industry requirements of the financial engineer.

Such calculator, is satisfying for mathematical, engineering, statistical disciplines, having an equation solver (by iteration) and a simultaneous equation solver. The calculator is useful starting with the teaching of algebra math to school students, and thus the calculator  
10 has a temporary disable feature on memory and/or graphics, for testing sessions. A separate hardware, complementing calculator is a central storage device, to which calculator or multiple calculators connect, to either load from device or store to device.

The calculator, having a resident, on-board memory, and providing an LCD screen and wherein having graphical display and rendering via screen or by output port. Memory includes  
15 short coded demonstrations of the calculator's functions and coded mathematical algorithms, demonstrating mathematical formulations, wherein graphing even as artwork. Such short coded demos in on-board or loaded by input port, engage interest in functions.

In addition to said above coded functions, the calculator features a reference compendium, an abbreviated dictionary and encyclopedia of mathematical, scientific and  
20 engineering terms, theorems, equations and thinkers, displaying such items on the screen.

General innovative functional aspects of the calculator, the Figures 95 through 99. Comprehensive coded financial engineering equations, reference resource items and sources, resident numerical processing and programmed calculating functions, Figure 100.

*Don Adams*